Determining the Speed of Sound in Air with the Doppler Effect

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Abstract

To determine the speed of sound, we measured the sound intensity detected from a 1000 Hz, constantly accelerating source (a car with an IoLab in the car to track this acceleration) in a stationary observer frame (a stationary IoLab). We implemented a Fourier transform algorithm to determine the sound frequency being picked up by the IOLab across the time interval of measurement. Then, upon plotting the frequency detected against the velocity of the observer (determined by integrating the acceleration plot) and parametrizing the resulting plot with the Doppler effect theoretical model, the speed of sound was determined to be 358.2 ± 7.4 m/s, which does not quite agree with the theoretical speed of sound, 343 m/s.

Introduction, Objectives, and Motivation

The purpose of this experiment is to experimentally determine the speed of sound, implementing the Doppler effect theoretical model. In implementing this model, we will also be able to verify the relationship it highlights between the observed sound frequency and observer velocity (with the stationary sound frequency and the speed of sound held constant).

Theory

The Doppler Effect:

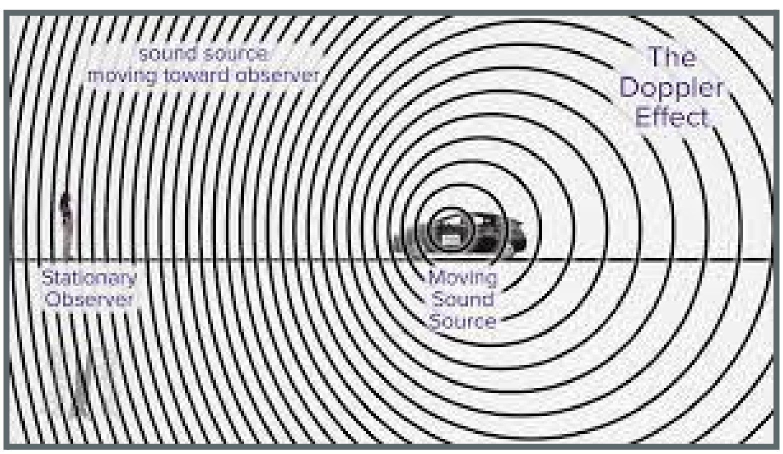


Figure 1: An illustration of the frequency shift in sound waves observed from a reference frame which the sound source is moving relative to.

The Doppler effect describes the phenomenon which takes place when a sound source and an observer are in motion relative to each other. There are two parts to this effect: the observer's motion and the source's motion. In this experiment, it is the source that is moving while the observer remains stationary. Therefore, this theory section will explore that portion of the theory.

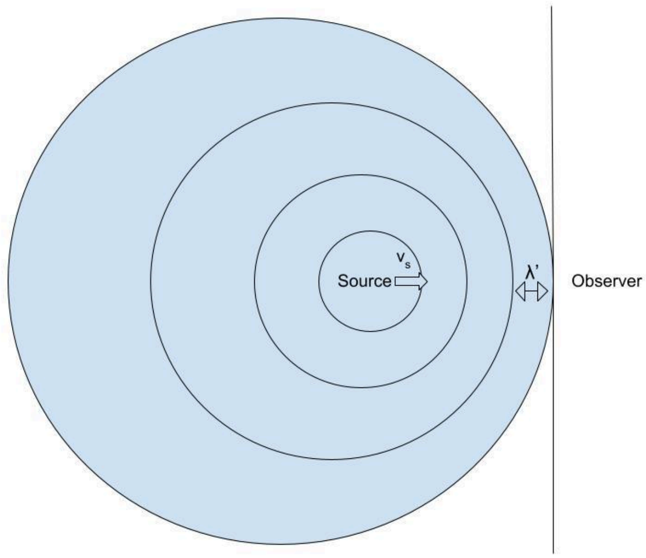


Figure 2: When the source moves towards the observer, the observed frequency (which depends on the intervals in which each wave passes the observer, represented by λ') increases. The opposite phenomenon occurs when the source moves away from the observer.

Let the wavelength of the sound waves relative to the moving source be λ and the frequency at the source be f_s. Then, the wavelength of the soundwaves relative to the stationary observer is:

$$\lambda' = \lambda - v_s/f_s$$
$$\Rightarrow f_o = v/\lambda' = v/(\lambda - v_s/f_s) = (v/\lambda)/(1 - v_s/f_s\lambda) = f_s/(1 - v_s/v)$$
$$\Rightarrow f_o/f_s = v/(v - v_s).$$

Or, if the source is moving away from the observer:

$$f_o/f_s = v/(v + v_s).$$

Thus:

$$f_o/f_s = v/(v \pm v_s) \quad \text{(Equation 1).}$$

In this experiment, we are interested in determining the speed of sound from the measured observed frequencies and source velocities across time (the source frequency will be held constant). Therefore, the following relationship between the source velocity and observed frequency derived from Equation 1 will be useful:

$$v_s = v \cdot |1 - \frac{f_s}{f_o}|$$

Equation 2.

Design and Methods

Equipment:

- Car
- iOLab x 2 (Respective computers)
- Speaker
- Smartphone (with the Frequency app)
- Tape
- An empty and free to use road of around 100 m

Procedure:

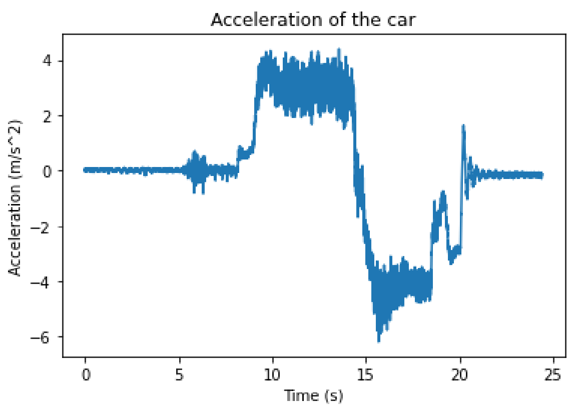
For the setup for this experiment, secure one iOLab measuring acceleration in the car with one of the axes pointed in the direction of motion. Similarly, secure the speaker to the car as well and connect the speaker via bluetooth to the smartphone and begin playing a 1000 Hz frequency tone. Around 50 m away from the car, set up the other iOLab (observer) to measure audio via the microphone sensor. Ensure that the microphone is recording at 4800 Hz rather than 2400 Hz. To collect data for this experiment, begin recording on both iOLab devices and start accelerating towards the observer and decelerate once passing and safely come to a stop. Stop recording on both iOLab devices and repeat the previous data recording steps for multiple trials.

There were a few design choices that were made to streamline and optimize the data collection process. The original procedure involved having one iOLab in the moving frame (in the car) and having it measure both the audio and acceleration data, however this was scrapped for the current set up for 2 reasons. One iOLab device cannot simultaneously measure audio and acceleration data and placing the microphone sensor in the car made it difficult to hear the tone due to overwhelming wind. Due to these limitations we employed the use of 2 separate iOLabs.

Analysis

Data Reduction:

The raw data from the iOLab devices consists of an acceleration over time plot and an audio intensity overtime plot. We identified the trial with the least jitter and noise in both plots. For our purposes this was the first trial.



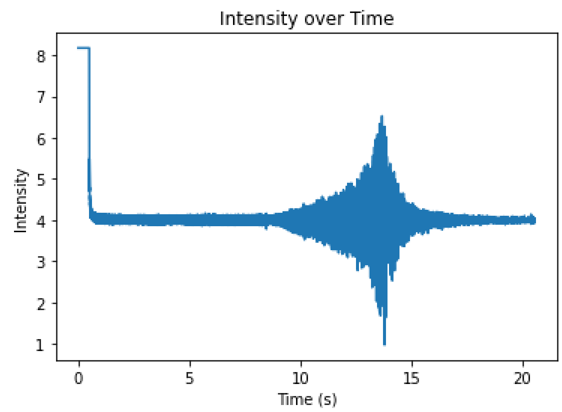


Figure 3, 4: Plots from the accelerometer and microphone sensors of the iOLabs respectively

The first step of the data reduction was to numerically integrate the acceleration data to find the velocity of the moving frame over time.

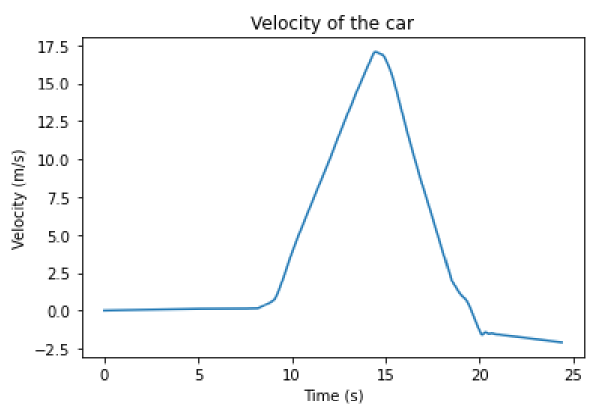


Figure 5: The velocity over time plot integrated from the accelerometer data

To begin the data reduction of the audio data we performed a Fourier transform via the matplotlib specgram method and mainly default parameters with a sample frequency of 4800 Hz. This provided the intensities of frequencies in the audio data over time. We then align the velocity and audio data so that they both have the same time steps. Due to the poor frequency resolution of the iOLab which can be seen on the vertical lines in Figure 6 we manually selected data points on the spectrogram of the frequency measured by the observer from the speaker, shown as the red data points. These were then matched to the velocity at that time to create a plot of the measured frequency against the velocity at that time.

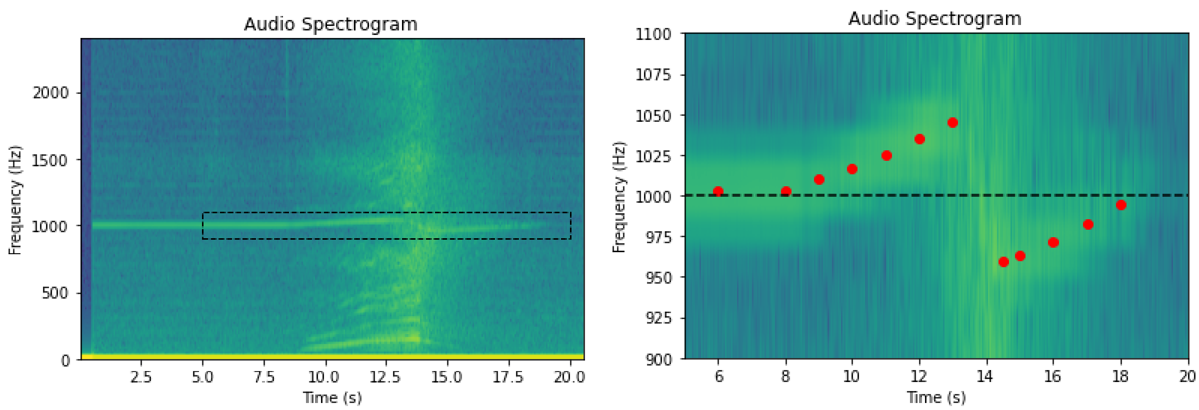


Figure 6, 7: The audio full spectrogram, The audio spectrogram of the relevant portion of the audio data within the relevant frequency and time range (boxed in the full spectrogram)

Fitting and Analysis:

The frequency vs velocity curve was then fitted with Eq. 2 where the speed of sound is the lone parameter. We did not include a vertical offset parameter because we believe that the frequency was close enough to 1 kHz when the velocity was 0 m/s.

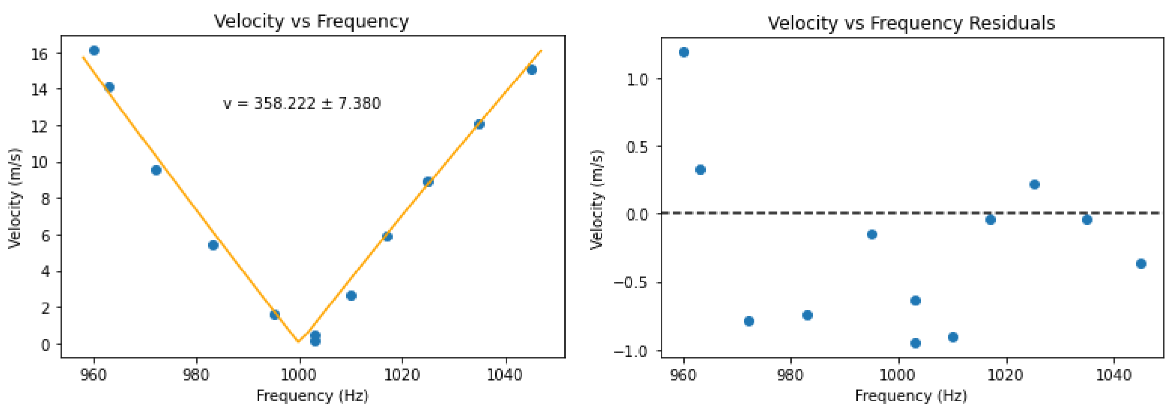


Figure 8, 9: The frequency vs velocity plot with the best fit model, The corresponding residual plot

This fit resulted in a value for the speed of sound of 358.2 ± 7.4 m/s. This does not quite agree with the theoretical value for the speed of sound at 20°C of 343 m/s. We also believe that the fit of the model was reasonable because we cannot identify any structure in the residuals. Despite our value not fitting the actual value for the speed of sound we believe this was a successful experiment in confirming the Doppler effect in air.

The primary sources of error in this experiment are due to the poor frequency resolution of the iOLab microphone sensor. This forced us to manually select data points to conduct the fit and reduced the number of data points we could fit to. Both of these introduced a significant source of error in our analysis. This can be seen especially at higher velocities as the frequency shift was even more difficult to determine.

Summary and Conclusion

The purpose of this lab was to experimentally determine the speed of sound implementing the Doppler Effect Model of Equation 1. Although the experimentally determined value for the speed of sound did not agree with the theoretical value, the Doppler Effect model was consistent with the experimental results (indicated to us by the randomness in residuals).

For this experiment, we measured the acceleration of a car with a speaker playing a 1 kHz tone as it drove past an observer recording audio data. Using this data we calculated the car's velocity and the change in the frequency of the tone and plotted a relationship between the 2 to determine the speed of sound in air. Our analysis showed that we determined the speed of sound to be 358.2 ± 7.4 m/s. This does not quite agree with the actual speed of 343 m/s however the residuals do not show any significant modeling error. This leads us to believe that the source of the error is elsewhere, primarily in the poor frequency resolution of the iOLab microphone. This limitation led to a smaller number of data points and an imprecision of those data points.

We would have been able to measure a more accurate and precise speed of sound if we had access to a microphone sensor with a higher sample rate. This would give us a better frequency resolution on the spectrogram. This would improve our calculations two-fold: it would allow us to automate the process of finding the frequency shift at different times on the spectrogram (which is currently being done manually) and it would allow us to have more data points because we are able to fit more points to the frequency.

Overall we believe that this was a successful experiment that demonstrated the Doppler effect and the feasibility of measuring the speed of sound in air using a similar setup. As previously mentioned, any future attempts of this would require a larger road to conduct the experiment on and a microphone sensor with better precision. Both of these improvements would provide more data points and a better fit, leading to a more accurate value for the speed of sound.

References

Doppler Effect Theory:

The doppler effect theory was reviewed from this website: LINK

Appendix

Data:

The data sets used for this experiment can be found here: LINK

Python Script:

The python script used for the data reduction and analysis can be found here: LINK