Experimental Determination of the Speed of Light

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Saahit Mogan, Johan Vonk, Samyak Tiwari

Abstract

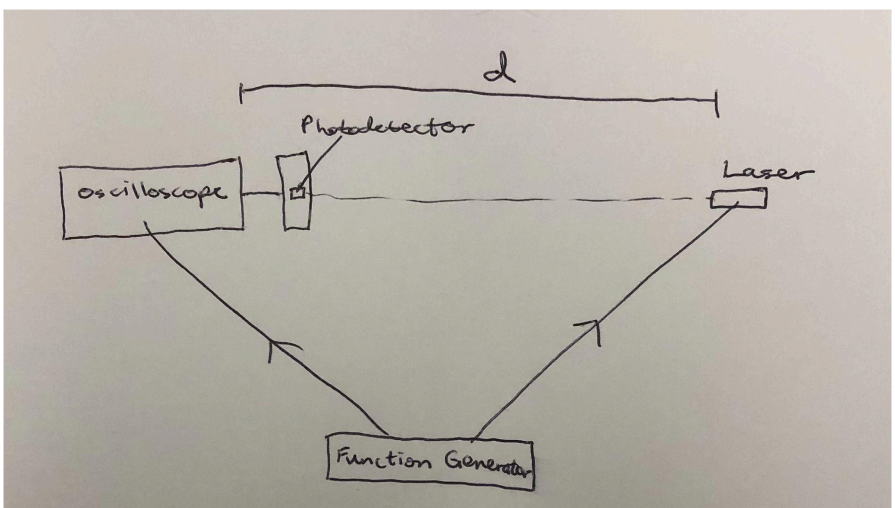
The wavelengths and frequencies of three monochromatic laser beams were measured. Wavelength was experimentally calculated by observing the interference pattern formed by the rays of a split beam to determine the modulated wave number difference between their paths. The red laser's wavelength was measured at 664 ± 24 nm, the green laser was measured at 563 ± 11 nm, and the ultraviolet laser was measured at 426 ± 14 nm. Only the green laser did not pass an agreement test with its corresponding theoretical wavelength. Next, the frequency of these beams were determined by shining them onto the photocathode, and finding the voltage at which all current was stopped (a known world function for the photocathode was necessary for this calculation). Finally, the fitted slope of a wavenumber vs frequency plot yielded the speed of light: c = (3.33 ± 0.05) x 108 m/s. Though this does fail an agreement test with the theoretical, it is close enough that it indicates experimental validity.

Introduction + Considered Methods

Our goal is to measure the speed of light in a vacuum, an important fundamental constant in all reference frames. It is essential for everything from the definition of the meter to radar and GPS.

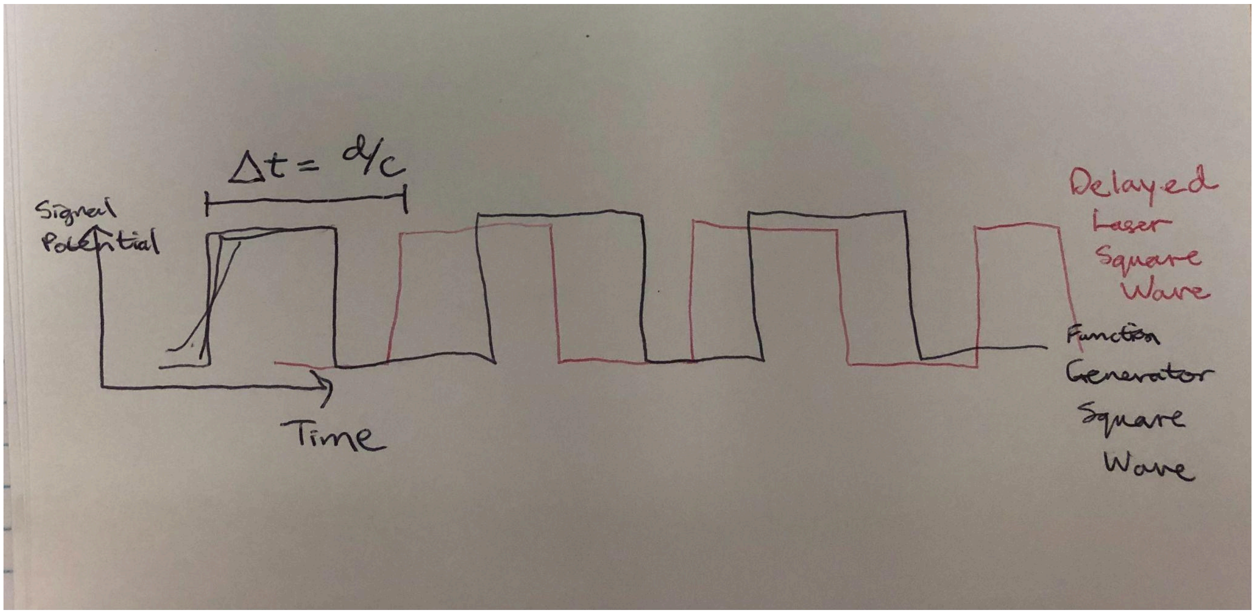
Initially, we were most intrigued by methods to make direct measurements of the speed of light. There were three we primarily considered: setting up an oscilloscope-function generator-laser circuit, the Fizeau apparatus, and the Foucault-Michelson apparatus.

<u>Setup 1: Oscilloscope-Function Generator-Laser Circuit:</u>



An image depiction of setup 1. By observing the time it takes for light to traverse a distance, d, we can make an experimental calculation of the speed of light.

In this setup, a function generator produces a square wave signal for an oscilloscope and a laser. The laser's response to that signal, whose oscilloscope read is delayed by the distance between the laser and the oscilloscope, will be viewable on the oscilloscope as a similar wave shifted to the right. In theory, the wave displacement should indicate the time it took light to traverse the distance, d.

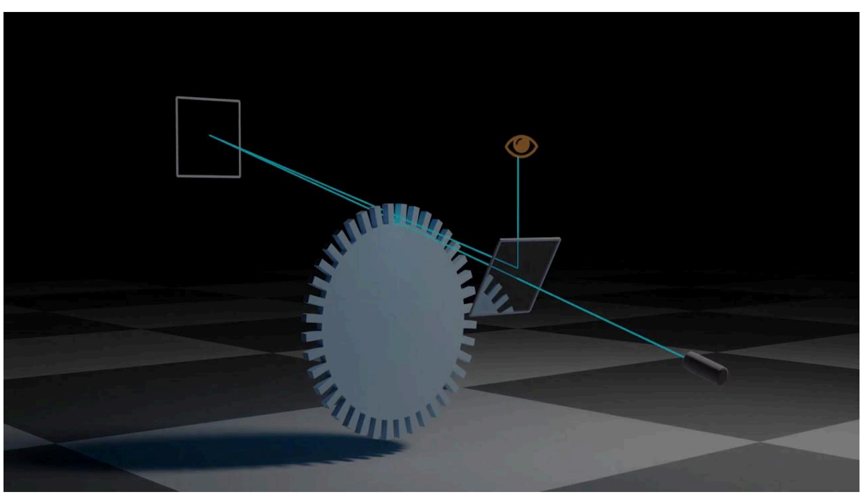


An ideal depiction of the oscilloscope output in this experiment is shown above.

In practice, however, the photodetector we used produced a lot of noise in its signal production. Too much noise to take any meaningful oscilloscope reading (with and without having the light run through a fiber-optic cable).

<u>Setup 2: Fizeau Apparatus:</u>

From the beginning, our considerations of this experimental setup were mostly for the sake of thought experiment (and provided a good segue to the final setup).



Fizeau Apparatus

The key to this setup is synchronizing the return travel of a beam of light bounced off a mirror with the rotation of the wheel so that any light rays occurring on the far mirror have their travel interfered by the next tooth. Observation of the angular velocity at which this synchronization occurs can yield necessary data to calculate the speed of light. The issue is, replicating this experiment would require a lot of space (on the order of kilometers) which we don't have in the lab.

Setup 3: Foucault-Michelson Apparatus:

The Foucault-Michelson Apparatus was the final setup we considered. The key idea of this apparatus is that when a light beam is shot through it, it will return precisely along the path that it traversed. This way, the light can be measured at a single point (the measuring microscope). However, if the mirror M_R is rotating, then in the time that the light travels to and from the fixed mirror, the angle of reflection will have changed slightly. As a consequence, a slight displacement can be observed between the incidence point of the outgoing beam and the incidence point of the returning beam. This can be used to determine the angular displacement which occurred in M_R in the time that it takes for light to go from the rotating mirror to the fixed mirror and back, and in turn the speed of light.

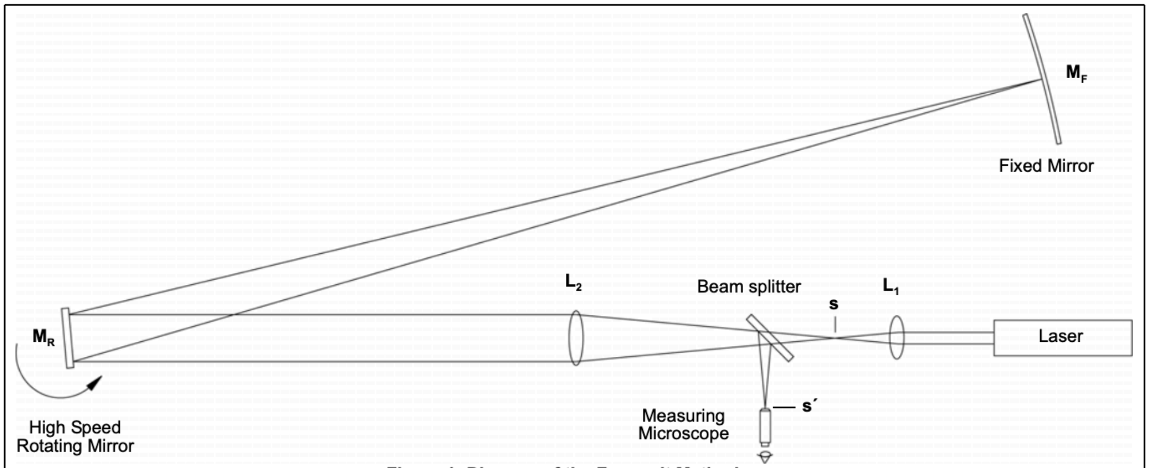


Figure 1: Diagram of the Foucault Method

While this was the closest setup we had to making a direct measurement of the speed of light, we couldn't get a mirror rotating fast enough to make it work.

Theory

The general theory for this experiment is quite trivial:

$$c = \lambda * f.$$

The speed of light can be determined by the product of wavelength and frequency. Alternatively:

$$f = 1/\lambda * c \qquad \text{(Equation 1)}$$

Thus, if we are able to construct a frequency vs wavenumber plot, the slope will yield a calculation of the speed of light constant!

More theory with regards to the calculations of wavelength and frequency is reviewed in each experiment.

Experiment 1: Wavelength

Theory

By pumping air out of a gas chamber with a pressure up to 76cmHg (at which point we'd have a vacuum), we cause a change in the wavenumber traversed across that chamber. Specifically,

$$m_{chamber} = 2dn_{chamber}/\lambda.$$

Specifically, we are concerned with the wavenumber difference between two paths: one through which the beam only travels through air and the other through which it travels through a chamber.

$$\Delta m = 2d(n_{air} - n_{chamber})/\lambda \qquad \text{(Equation 2)}$$

We can compute $n_{chamber}$ by treating the chamber as some proportion of air (1 - P/76) and the rest vacuum (P/76), where P is the pressure used to evacuate our chamber. This yields:

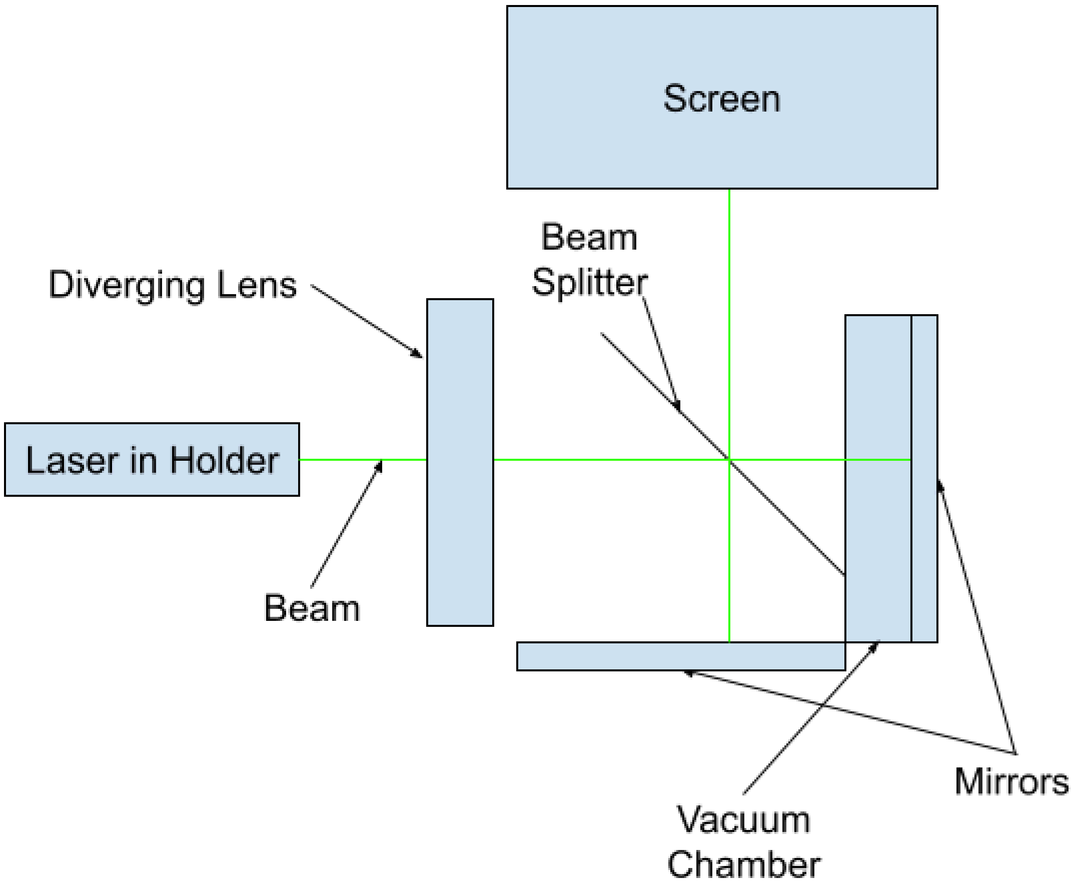
$$n_{chamber} = n_{air} + (1 - n_{air})P/76.$$

Combining this result with equation 2 yields our final result, a relationship between wave number offset and chamber pressure, and a slope dependent on our unknown, the laser's wavelength.

$$\Delta m = 2d/\lambda * (n_{air} - 1)P/76 \qquad \text{(Equation 3)}$$

Design

Our first experiment is designed to measure the wavelength of three different lasers using an interferometer and varying the optical path length along one of the paths. One design challenge was obtaining a large enough area on the screen so that we could see the refraction pattern. In our initial design, we did not have any lenses in the entire setup. In this case, the interference pattern on the screen was too small to observe clearly. Therefore, we first tried placing a diverging lens between the interferometer and the screen thinking that it would increase the size of the image. Unfortunately, this was not successful because the lens simply amplified the noise due to atmospheric refraction. Instead, we placed the lens between the laser and the interferometer which resulted in a clear image on the screen.



Top down block diagram of final experimental setup for Experiment 1

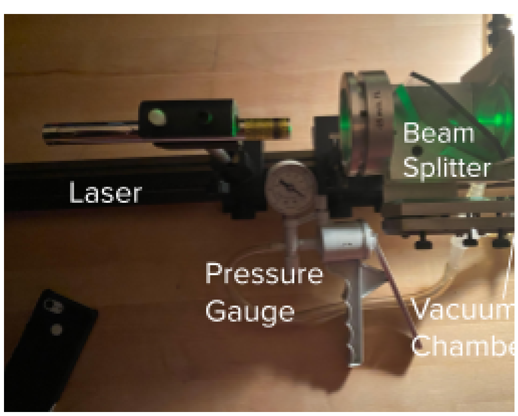


Photo of top down final experimental setup

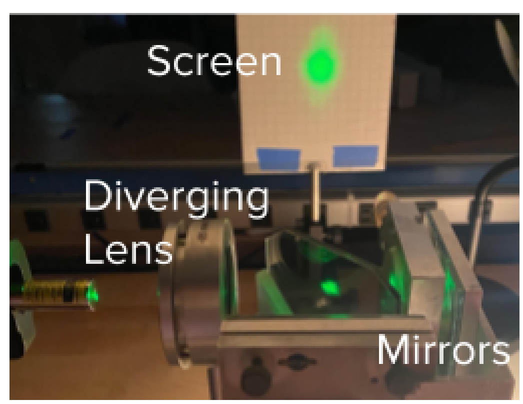
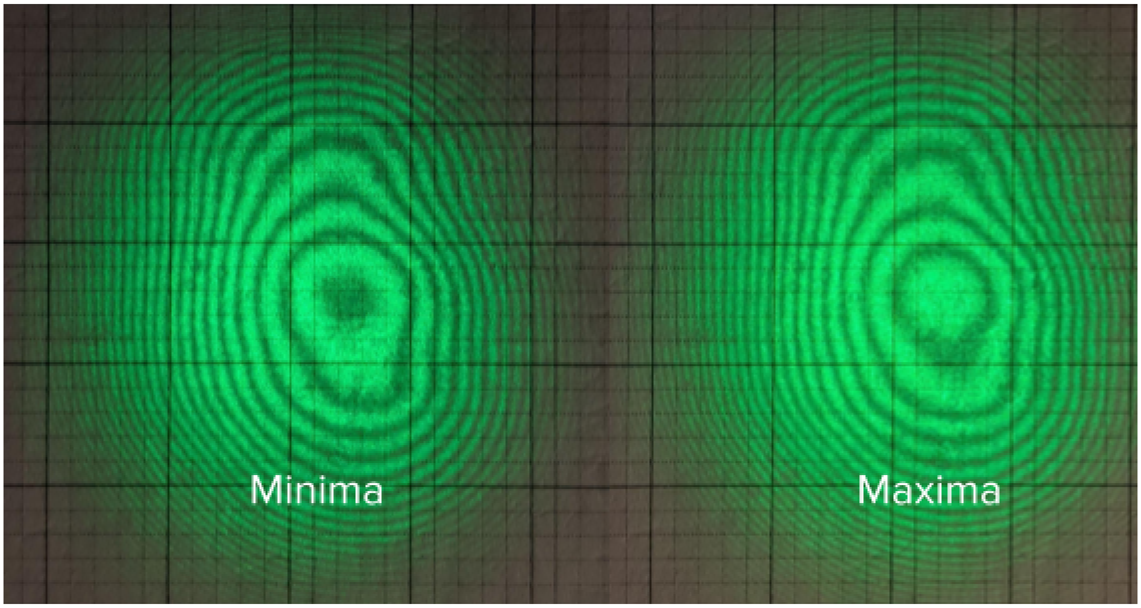


Photo of experimental setup viewing screen



Photograph showing interference patterns for minima (left) and maxima (right)

Methods

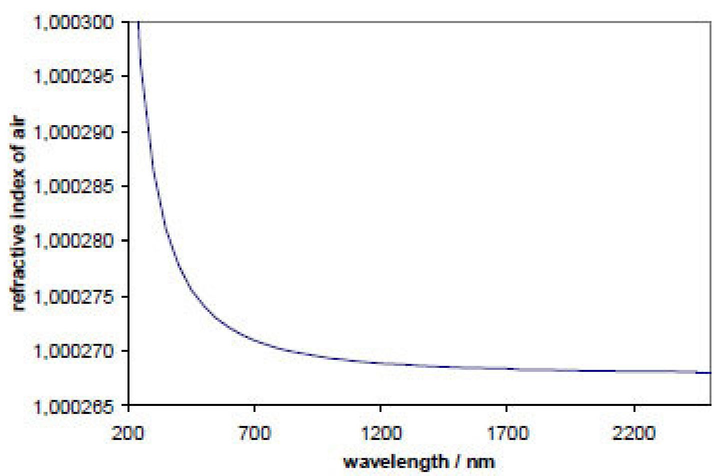
For each of the three lasers, we did the following:

1. Place the interferometer in the optical stand with one of the mirrors perpendicular to the optical axis.
2. Place the screen in an optical stand parallel to the other mirror.
3. Place the laser in the laser holder with one screw over the start button.
4. Alight the laser with the center of the mirror and then with the center of the screen.
5. Place a diverging lens between the laser and the interferometer on the optical axis.
6. Realign the laser so it is still centered on the screen.
7. Place the pressure chamber with no vacuum pressure in the path of the laser between the half-silvered mirror and the mirror perpendicular to the optical axis.
8. Adjust the three screws on the back of the interferometer until the interference pattern is centered in the laser area and appears clearly as a minima.
9. For the first laser only, for calibration, take several trials of the starting pressure of the chamber increasing the pressure and then releasing between each trial.
10. Release the vacuum pressure from the chamber by letting air in.
11. While slowly increasing the pressure by pumping small amounts of air out of the chamber, count the number of times a minima is reached.

12. From the final minima estimate the fractional change to the next minima. A maxima would be 0.5 and values were estimated between these two options based on brightness, hence why there is an error of 0.5, since only the maxima, minima distinction is precise.
13. Add the whole and fractional change as the change in wavevector and record in the data table.
14. Record the pressure in the data table.
15. Repeat steps 10-14, 33 more times for 34 trials in total.

Preparation

Our full data table is in the appendix. Since all our data was in the required units for this experiment, we simply had to add the error using uncertainties. We also calculated the average calibration pressure and error as the standard deviation since it is used for each individual measurement. We combined the three datasets of the three lasers into one dictionary so we could write one function to process all three. We also subtracted the calibration pressure from each datapoint and combined the standard deviation in the data points with the standard deviation in the calibration pressure. We also used the formula from the theory section with a value for the index of refraction of air of 1.000293 at 20 degrees celsius and 589 nm.[3] Yet the index of refraction actually varies with wavelength.[4] Since we are trying to measure the wavelength, we cannot use this value without incorporating the exact equation into our model. In addition, this would require obtaining temperature, humidity, and CO_2 concentrations for maximum accuracy. This variation is significant and could contribute to errors. The table from[4] is shown below.



Fitting

We used orthogonal distance regression to fit a simple linear (zero intercept) model for the slope of each of the lasers. Then, we combined the uncertainty in the slope fit with the standard error of the individual points slopes calculated by dividing wavevector change by pressure directly. Then we plotted the resulting data with its error along both axes and the slope and the range of the slope plus or minus its combined standard deviation. We also plotted the residuals of the resulting fits. All six of the resulting graphs are shown below. All of our error propagation was done with the Python uncertainties module. In addition, while all our adjusted chi squared values are quite low, this is likely due to using 0.5 for the error in the change in wavevector, since while our estimate was qualitative it was better than rounding to the nearest value. The resulting error in the slope is not reliant on these errors, so it should be more accurate. The resulting graphs are shown below:

Figure 1: Red Laser - Change in Wavevector vs Pressure

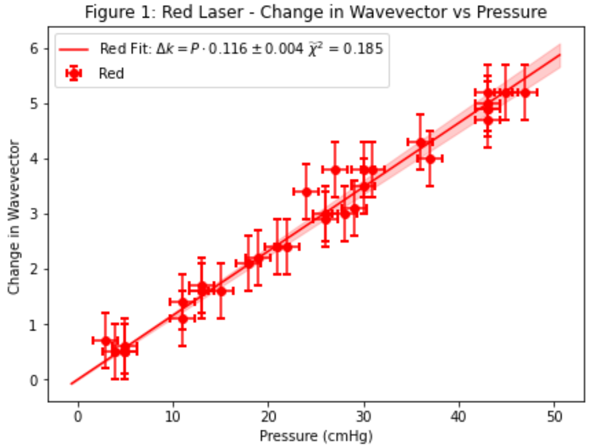


Figure 2: Red Laser - Change in Wavevector vs Pressure Residuals

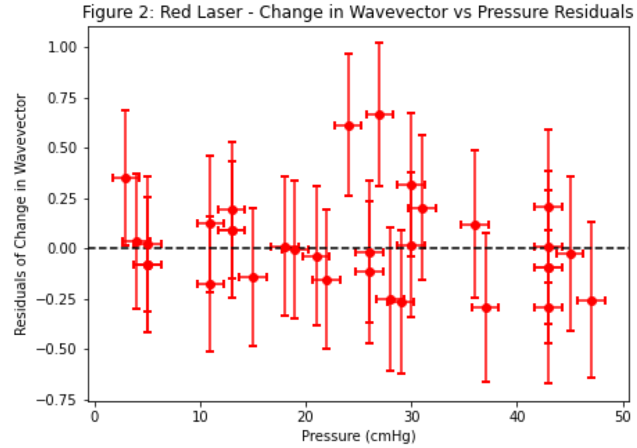


Figure 3: Green Laser - Change in Wavevector vs Pressure

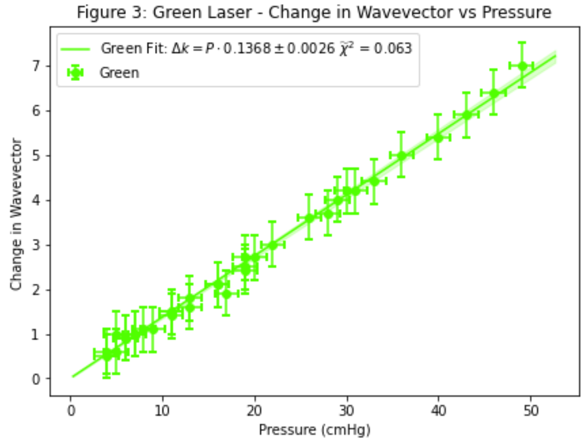


Figure 4: Green Laser - Change in Wavevector vs Pressure Residuals

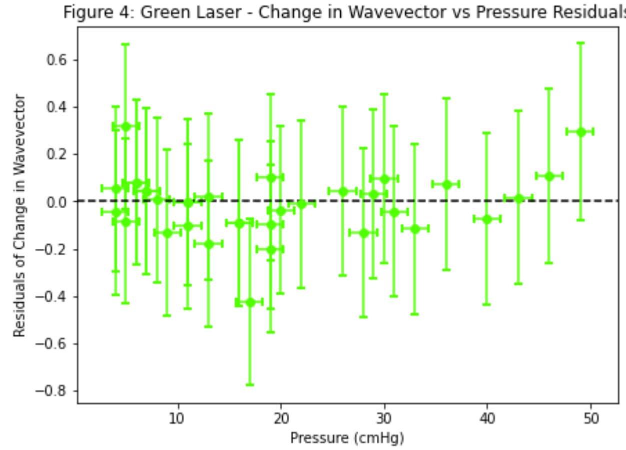


Figure 5: Ultraviolet Laser - Change in Wavevector vs Pressure

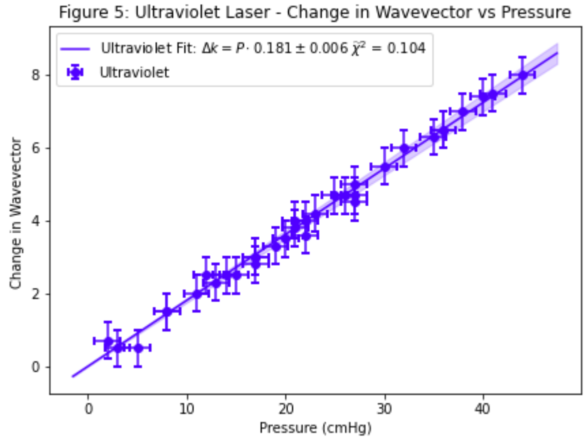
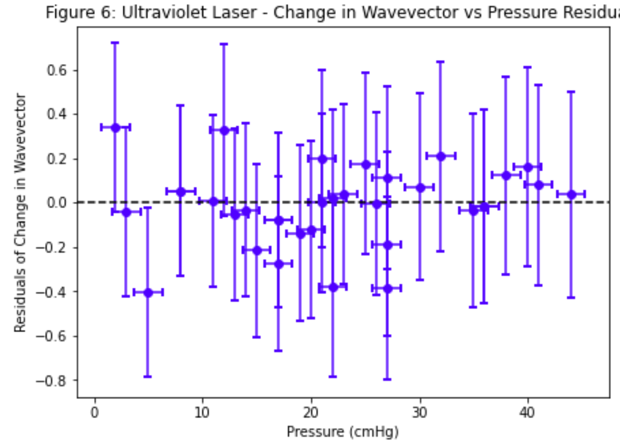


Figure 6: Ultraviolet Laser - Change in Wavevector vs Pressure Residuals



Analysis

In figures 2, 4, and 6 we see that the residuals for each of the lasers show no clear pattern. This means that the linear fit (without intercept) is a good representation of the underlying data. This makes sense because according to Equation 3, the change in wavevector and pressure should be linearly correlated. Plugging in our slope values to the formula and accounting for errors, we get values for the wavelengths of each laser.

The red laser is measured at 664 ± 24 nm, while it specifies a 655 ± 15 nm wavelength on the laser. Performing an agreement test, we get a value of 0.165 which passes. The green laser is measured at 563 ± 11 nm, while it is specified as a 532 ± 1 nm wavelength. Performing an agreement test, we get a value of 1.427 which fails. The ultraviolet laser measured at 426 ± 14 nm, while the laser specifies exactly 405 nm. Performing an agreement test, we get a value of 0.744 which passes.

The green laser fails its agreement test in isolation, since we are taking three different measurements there is a more than 5% chance that one fails. In fact, since we are testing the same apparatus, we are only interested in if all tests fail. By themselves, the red result has an agreement probability of 74%, the green result 0.4%, and the ultraviolet result 13%. Yet since we have three data points, we should multiply each of their standard deviations the square root of three. Thus the probabilities of each become 85% for red, 10% for green, and 39% for ultraviolet. Now we must multiply the three to get a probability of 3% which still fails but by a much narrower margin. This could be because either the laser wavelength errors are not accurate or because our measurement procedure is not calibrated correctly. Since we require the density of air to measure the change with the vacuum pressure, we assume the same standard temperature and pressure conditions that are used in the initial experiment to calculate the index of refraction of air. If we used a different gas that does not vary with temperature and pressure and whose index of refraction is known precisely, we could make more accurate calculations.

Conclusions

In conclusion, our wavelength results showed decent agreement with the theoretical values given by the laser manufacturers. Since we are using our calculations to calculate the speed of light, if we take our error into account, we should still be successful. In the future, we should fit with the exact equation of index of refraction of air with wavelength as part of our formula, but this fit is not possible without measuring temperature and would be improved by approximate humidity and CO_2 concentrations. In a future experiment, we could simply measure the temperature of the air that is leaving the vacuum pump to improve accuracy.

Experiment 2: Frequency

Theory

When a photocathode is excited by light above a certain frequency (known as the threshold frequency, f_0), photoelectrons are emitted from the material. This emission of electrons can then be measured as a current. We can determine the maximum kinetic energy of these electrons by measuring at what voltage the current drops to 0. This voltage (known as the stopping voltage, V_{stop}) is proportional to the maximum kinetic energy of the emitted photoelectrons by the charge of an electron. The kinetic energy of the photoelectron can also be determined with the equation:

$$K_{max} = h(f - f_0) = hf - \phi$$

where f is the frequency of the incident light and ϕ is the work function defined as hf_0. Using these 2 relationships we can create a linear relationship between the stopping voltage and the frequency of incident light which is given by:

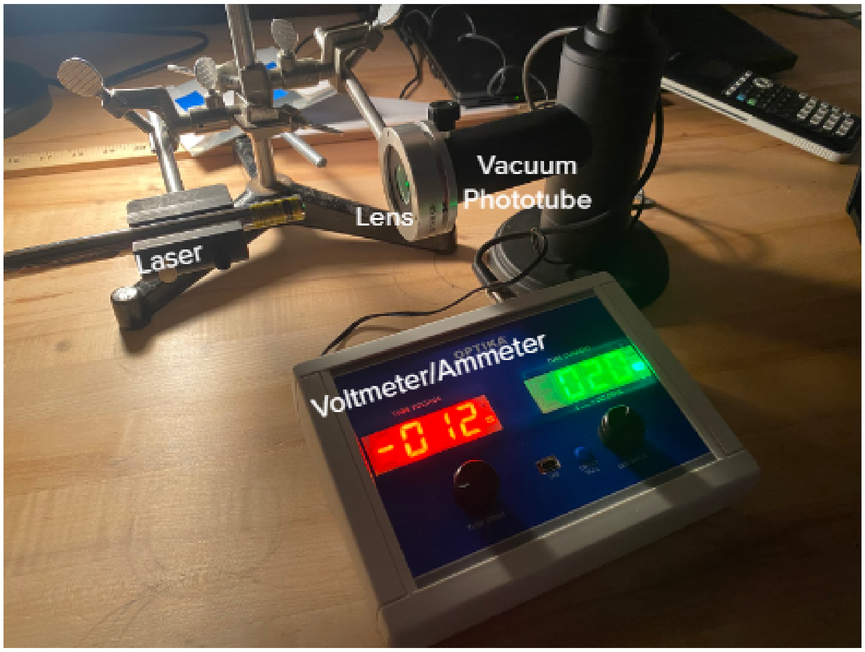
$$eV_{stop} = hf - \phi \quad \text{(Equation 4)}$$

Using a known work function, we can measure the stopping voltages to determine the frequency of the incident light.

Methods

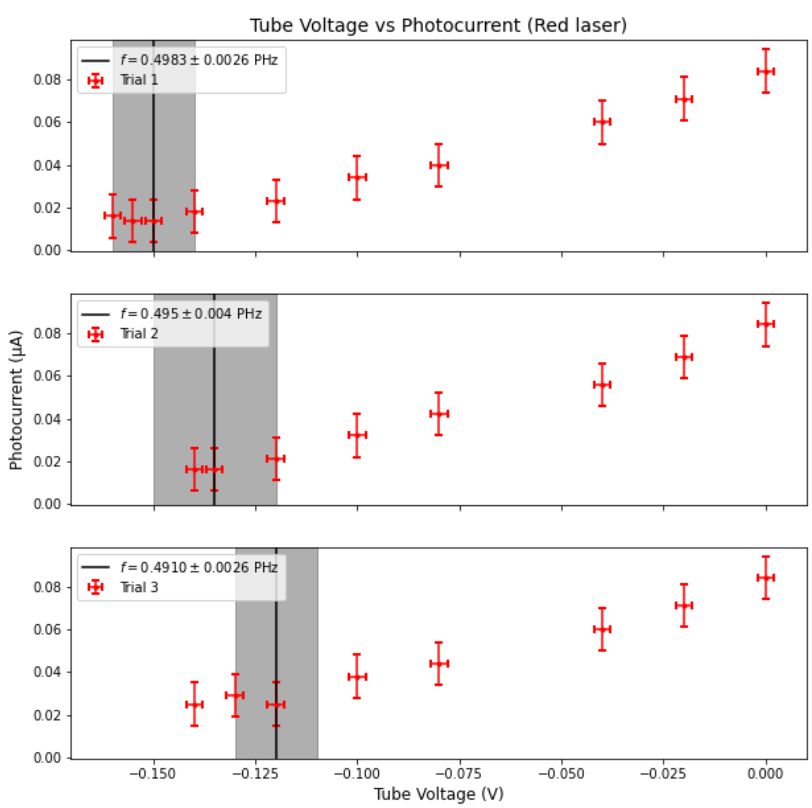
In this experiment, we set up the photoelectric effect apparatus. The laser was mounted to the stand as well as a diverging lens in front of it to spread out the light. The lens used in this experiment had a focal length of -25.0 cm. Once the lens was mounted and aligned with the lens and the apparatus, the lights were turned off and the voltmeter/ammeter was turned on and zeroed out. To eliminate sources of uncertainty we ensured that the laser was perpendicular to the table and that the laser was as close to the opening as possible. By limiting the gap between the laser and the opening we decrease the chance that outside light entered our experimental setup.

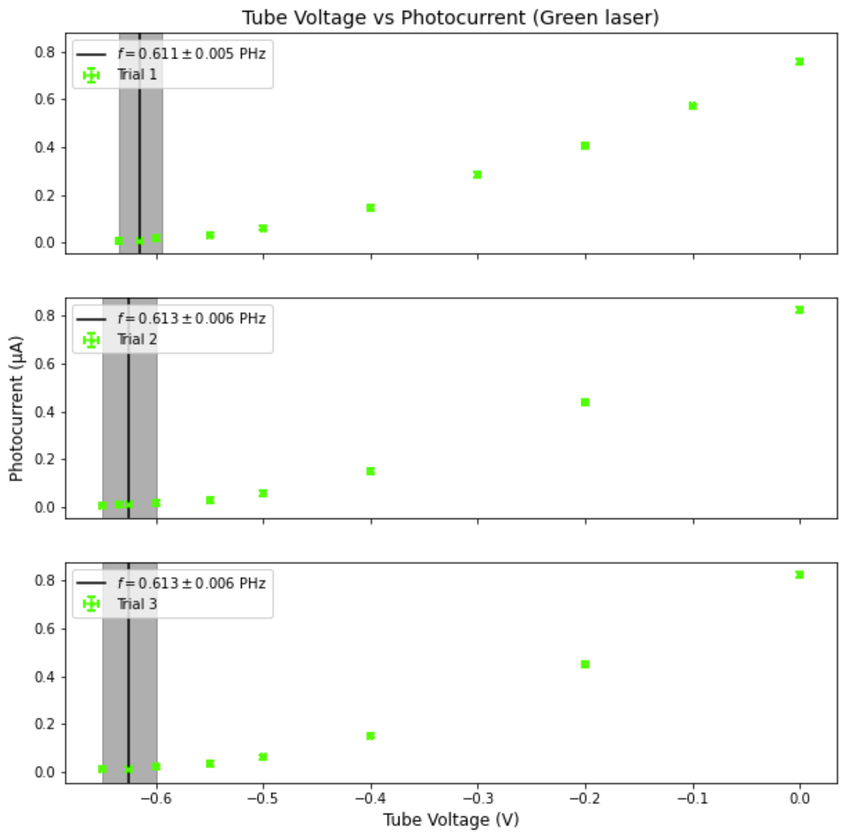
To begin the data collection, we measured the photocurrent across the circuit as we changed the anode voltage. The voltage once again started at 0 and was slowly decreased until the photocurrent became very close to or below 10 nA which was the precision indicated by the manual for the ammeter.. When it was close to 0 we decreased the size of the increments at which we measured the photocurrent. The stopping voltage is recorded as the lowest voltage at which the photocurrent no longer decreases. The precision error in our measurement was taken to be the increment at which we set the back voltage when we reached the stopping voltage. These measurements were done for each of the 3 lasers: red, green, and violet.

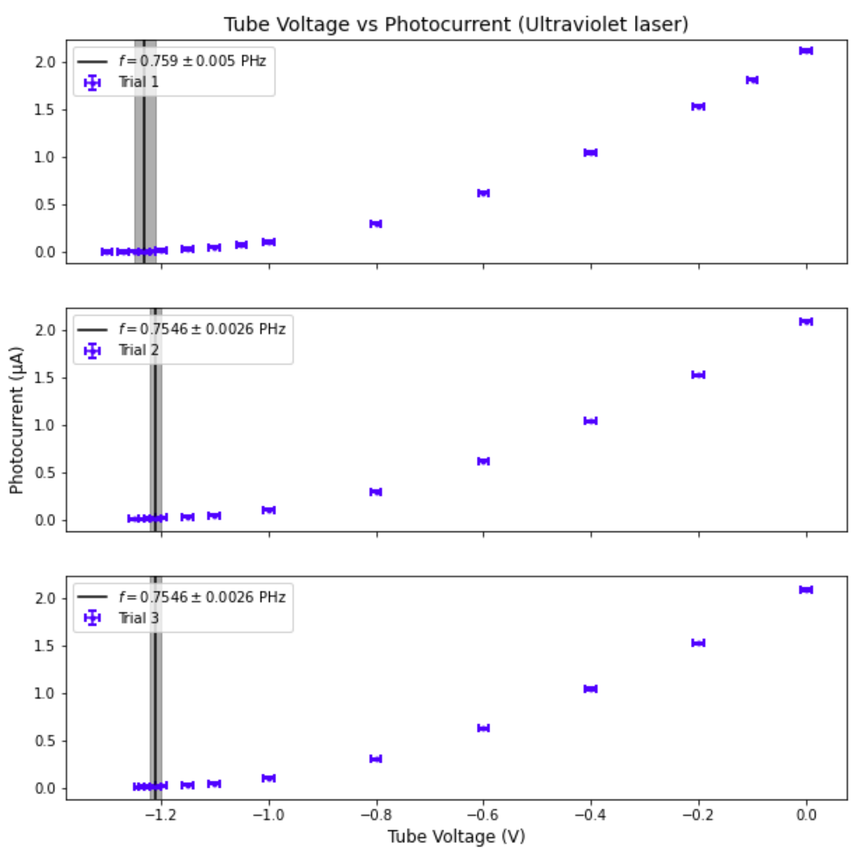


A labeled image of our experimental setup for the green laser. The setups were identical for the other laser colors.

Results




Tube Voltage vs Photocurrent (Green laser)
$f = 0.611 \pm 0.005$ PHz
Trial 1
$f = 0.613 \pm 0.006$ PHz
Trial 2
$f = 0.613 \pm 0.006$ PHz
Trial 3
Photocurrent (μA)
Tube Voltage (V)


Tube Voltage vs Photocurrent (Ultraviolet laser)
$f = 0.759 \pm 0.005$ PHz
Trial 1
$f = 0.7546 \pm 0.0026$ PHz
Trial 2
$f = 0.7546 \pm 0.0026$ PHz
Trial 3
Photocurrent (μA)
Tube Voltage (V)

Plots of the data collected from each of the laser colors indicated by the color of the data points. The actual stopping voltage is marked by the solid vertical line and the error is marked by the shaded region around the stopping voltage marking.

Laser Color	Measured Frequency (PHz)		
	Trial 1	Trial 2	Trial 3
Red	0.4983 ± 0.0026	0.495 ± 0.004	0.4910 ± 0.0026
Green	0.611 ± 0.005	0.613 ± 0.006	0.613 ± 0.006
Near-UV	0.759 ± 0.005	0.7546 ± 0.0026	0.7546 ± 0.0026

A table of the measured light frequency measured for each trial and for each laser color along with the standard error for each of the measurements.

Analysis

To test the results of this experiment we performed agreement tests on all of the measured frequency values. Unfortunately, all of the frequency tests failed with a value greater than 1. This may be due to 2 reasons: an underestimated standard error and/or an inaccurate measured frequency value. All of the measured values however were all greater than the expected value which may mean there was either a modeling error or systematic error we did not account for.

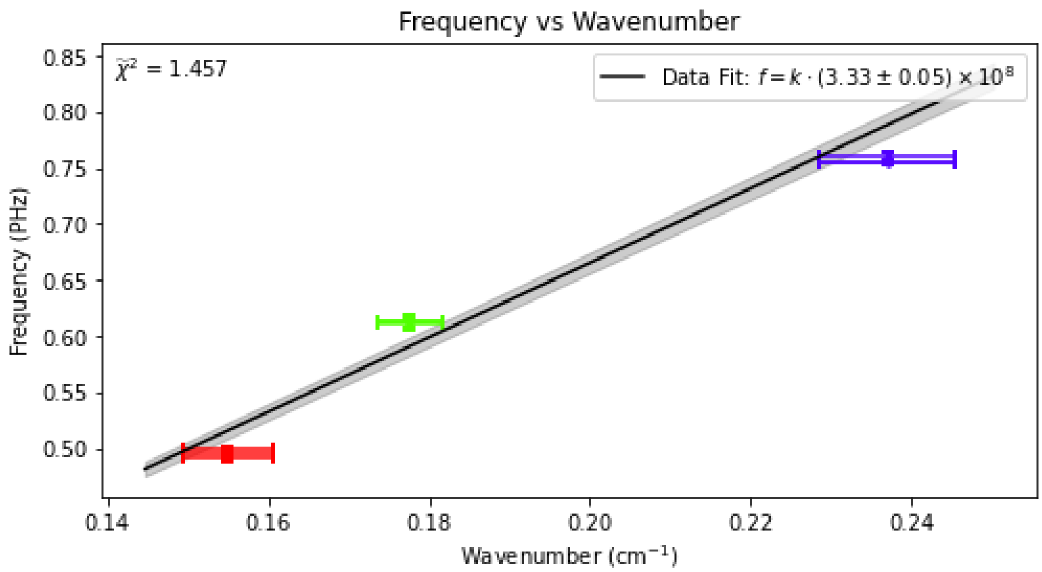
Since our errors were based on the smallest increment of the voltage measurement, we do not believe that the errors were the cause of the lack of agreement. We also accounted for essentially every case of stray light leaking into the vacuum phototube as well as zeroing out the ammeter when we are close to 0 nA on the photocurrent. This leaves the cause of our erroneous measurements to be a modeling error. We believe that the cause of this modeling error is from an incorrect work function value. This theory is further supported by the fact that during Lab 4 many groups derived a value for the work function that was lower than what was expected. A lower work function would result in lower frequency values which would result in more accurate value for our derived frequency values.

Conclusions

In conclusion, we were able to derive values for the frequency of each of the lasers. However none of our measured values agreed with the actual values for the laser frequency. We believe that the reason for this discrepancy was an inaccurate work function value which resulted in inaccurate frequency measurements. To account for this in the future, we could independently measure the work function of the vacuum phototube prior to the actual experiment. Another issue we encountered was an inconsistent intensity from the green laser, this was difficult to mitigate and just had to be dealt with. To hopefully improve this experiment further in the future we would use higher quality laser light sources such that we would have cleaner and more consistent light. Overall, despite the values not statistically agreeing they were still close to the theoretical values of the laser light frequency and we were able to achieve our goal of determining the frequency of our light sources.

Analysis + Conclusions

We have now experimentally determined wavelengths (1/wavenumber) for each monochromatic light, and we have three calculations for each light's frequency. This yields 9 points which we can plot to determine a wavenumber vs frequency relationship.



As explained by Equation 1, the slope of this relationship is the speed of light! Therefore, our final experimental calculation of the speed of light is 3.33 +/- 0.05 * 10^8 m/s. Unfortunately, this does not agree with the theoretical value 3.00 * 10^8 m/s by our standard agreement test. However, the closeness of these values speaks to the validity of the experimental setups implemented.

The final overarching analysis to be done here entails answering the question: why was our slope off? There are two key reasons for this. Firstly, it seems that the green laser's points are not along the same line as the red and violet lasers' points. Specifically, either the frequency was overestimated or the wavenumber was underestimated for the green laser (moreso than they were for the red and ultraviolet lasers). Judging from the fact that the work function appeared off in our Experiment 2, the former is more likely the case. Secondly, there seems to be the same error in the red and violet lasers (since the above plot's slope was an overestimation of the speed of light) to a lesser extent. Again, we believe that this systematic can largely be attributed to the work function's inaccuracy in Experiment 2 (while there was error in Experiment 1 as well, it seemed more random than sysematic). Though it is difficult to draw much from the fact that one laser (green) is offset from the line formed by the other two, it is important to note that the green laser was observed to have a flickering intensity, which may have caused this.

References

1. https://uncertainties-python-package.readthedocs.io
2. https://en.wikipedia.org/wiki/Speed_of_light#History
3. Zajac, Alfred; Hecht, Eugene (18 March 2003). *Optics, Fourth Edit*. Pearson Higher Education. ISBN 978-0-321-18878-6.
4. Jürgen Hartmann, "Correct consideration of the index of refraction using blackbody radiation," Opt. Express **14**, 8121-8126 (2006)
5. …

Appendix

Data Table for Experiment 1

Red Laser		Green Laser		Near-UV Laser	
Pressure (cm Hg) ± 1	Change in Wavevector ± 0.5	Pressure (cm Hg) ± 1	Change in Wavevector ± 0.5	Pressure (cm Hg) ± 1	Change in Wavevector ± 0.5
16	1.7	10	1	25	4
34	3.8	14	1.5	8	0.5
21	2.1	19	2.1	6	0.5
6	0.7	22	2.5	14	2
14	1.4	25	3	16	2.3
29	3	29	3.6	20	3
39	4.3	34	4.2	24	3.8
46	5.2	7	0.6	28	4.7
8	0.5	10	1	35	6
7	0.5	14	1.5	33	5.5
18	1.6	16	1.8	44	7.5
27	3.4	19	2.1	17	2.5
30	3.8	9	0.9	26	4.2
25	2.4	8	1	11	1.5
8	0.5	33	4.2	20	3
16	1.6	39	5	18	2.5
24	2.4	43	5.4	24	4
33	3.8	46	5.9	29	4.7

50	5.2	49	6.4	30	4.7
16	1.6	7	0.5	22	3.3
14	1.1	8	0.6	23	3.5
46	4.9	9	0.9	38	6.3
32	3.1	11	1.1	39	6.5
8	0.6	12	1.1	41	7
22	2.2	14	1.4	43	7.4
29	2.9	16	1.6	47	8
31	3	20	1.9	39	6.5
40	4	22	2.4	5	0.7
46	4.7	23	2.7	11	1.5
8	0.5	32	4	15	2.5
46	4.9	36	4.4	20	2.8
48	5.2	31	3.7	25	3.6
33	3.5	52	7	30	4.5
46	5	22	2.7	30	5