# Effects of Earth's Rotation on GW Signal Parameter Estimation

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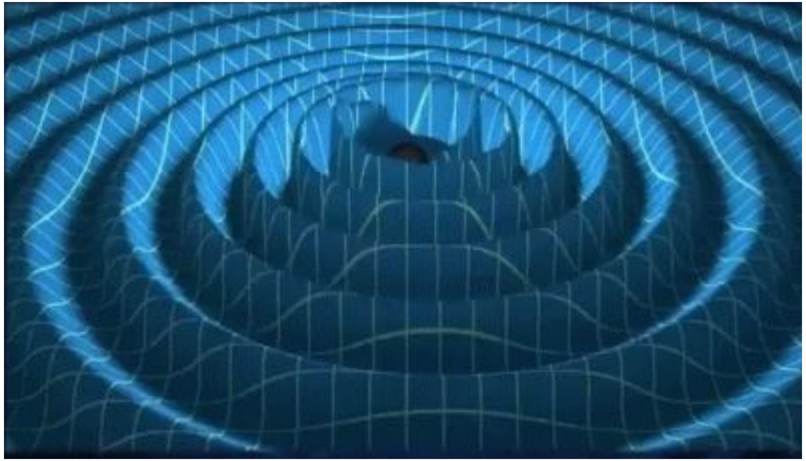
# Gravitational Waves

Solution to **linearized, source-free** EFE of the form:

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

Spacetime metric   Minkowski   Perturbation

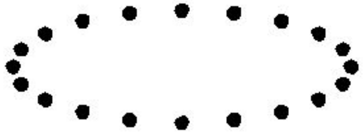
**Plane waves** oscillating through spacetime, produced by **compact object mergers**:

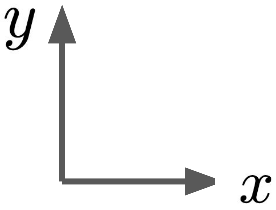


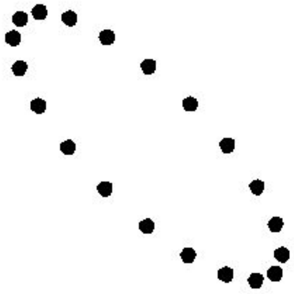
# Polarization States

$$h_{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & h_+ & h_\times & 0 \\ 0 & h_\times & -h_+ & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

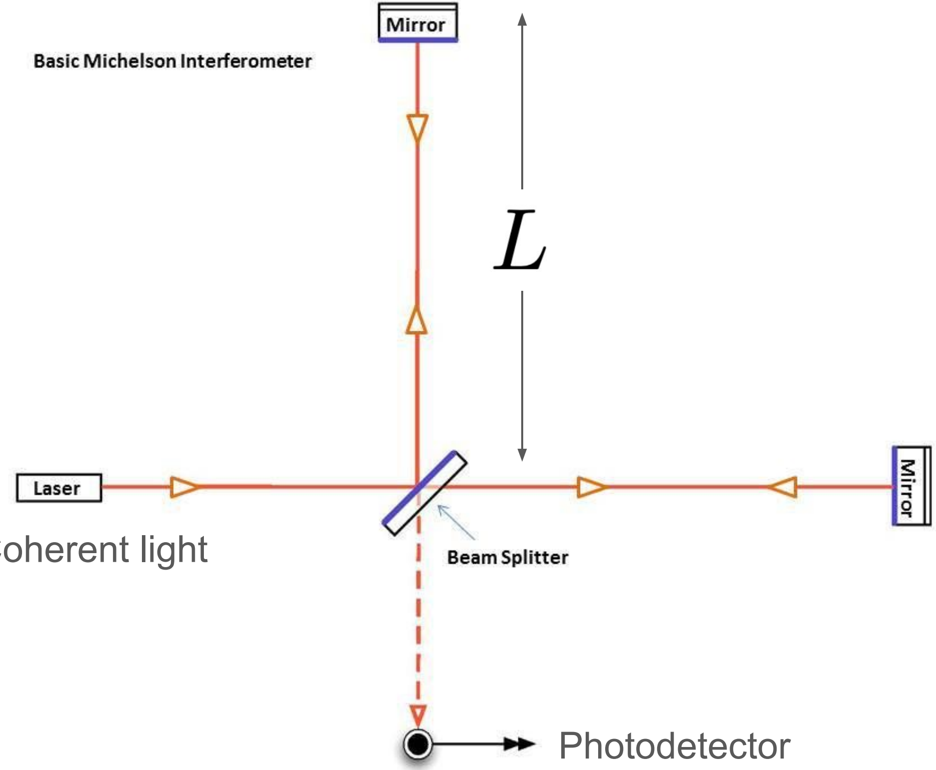
Independent + and × polarization states

Plus: 




Cross: 

# GW Detectors and the Strain




Coherent light

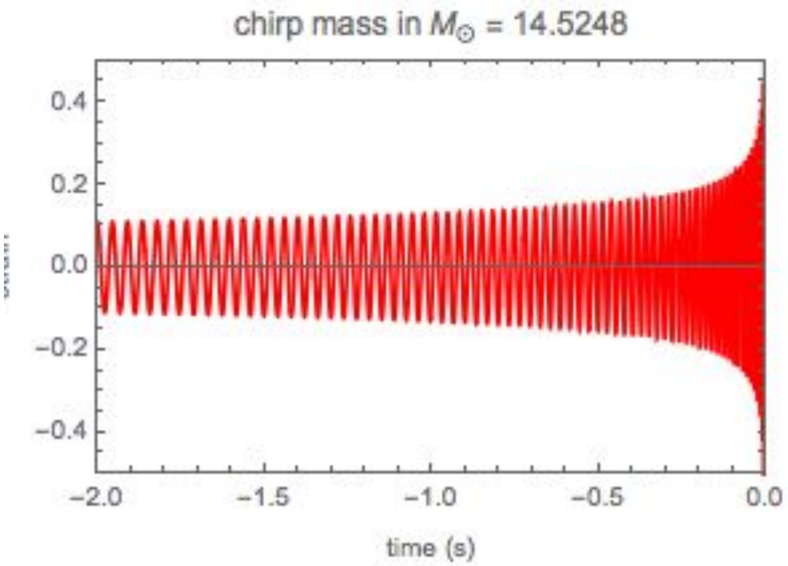
Photodetector

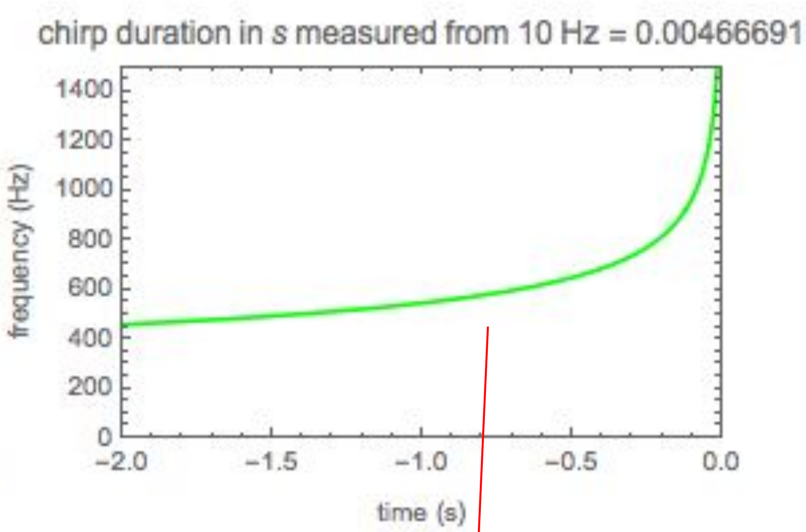
$$h = \frac{\Delta L}{L}$$

Strain

Relative change in arm length

# The Frequency of Our Signal

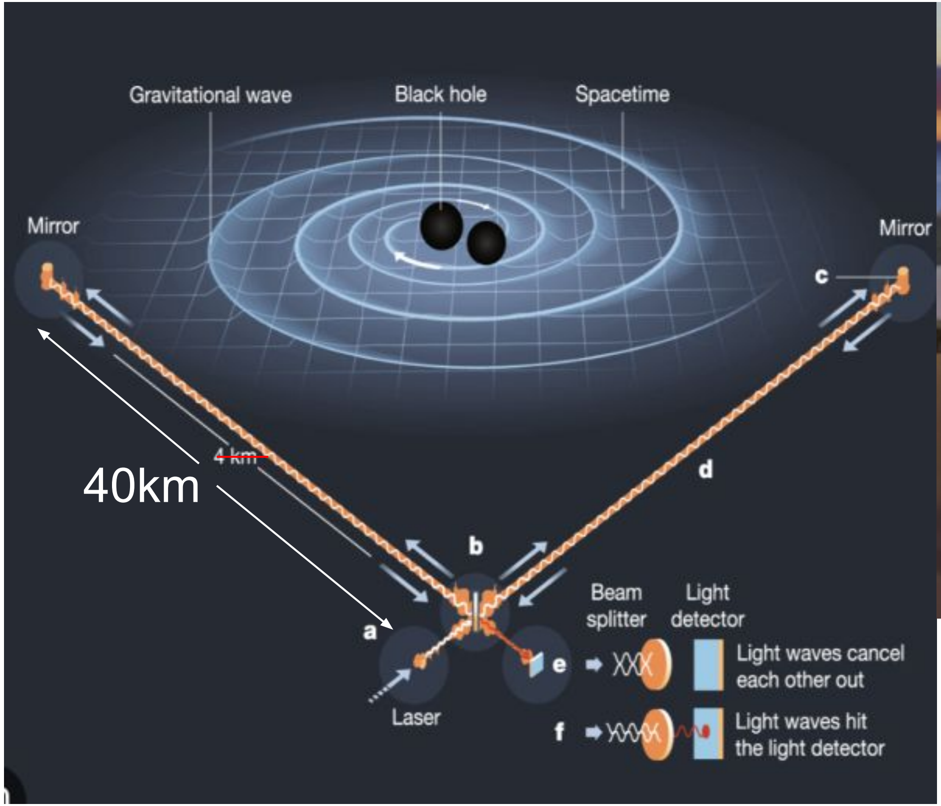


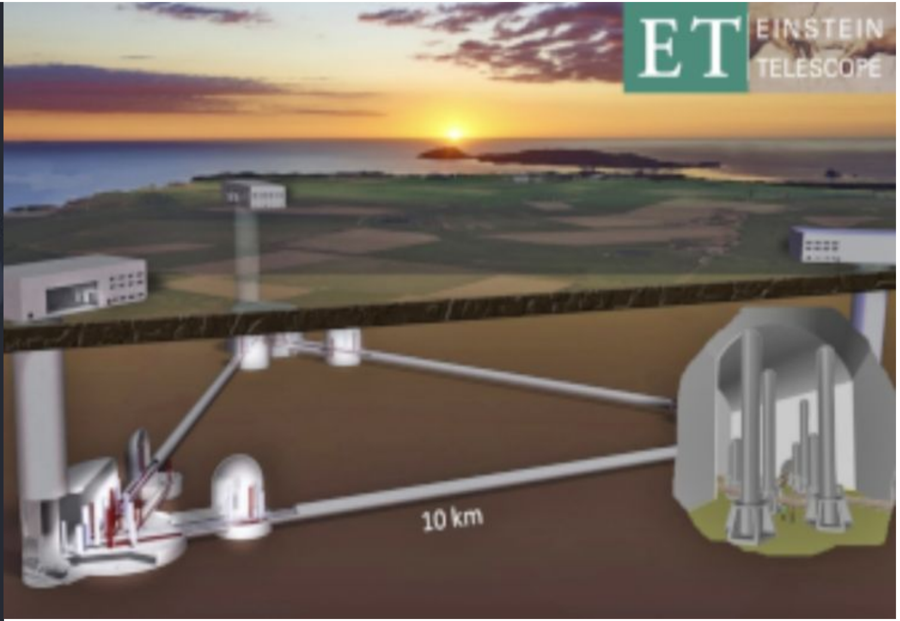




Larger than exponential growth!

$$2\dot{f}_{\mathrm{GW\,only}} = \frac{96}{5}\,\pi^{\frac{8}{3}}\left(\frac{G\mathcal{M}}{c^3}\right)^{\frac{5}{3}}\,(2\,f)^{\frac{11}{3}}$$

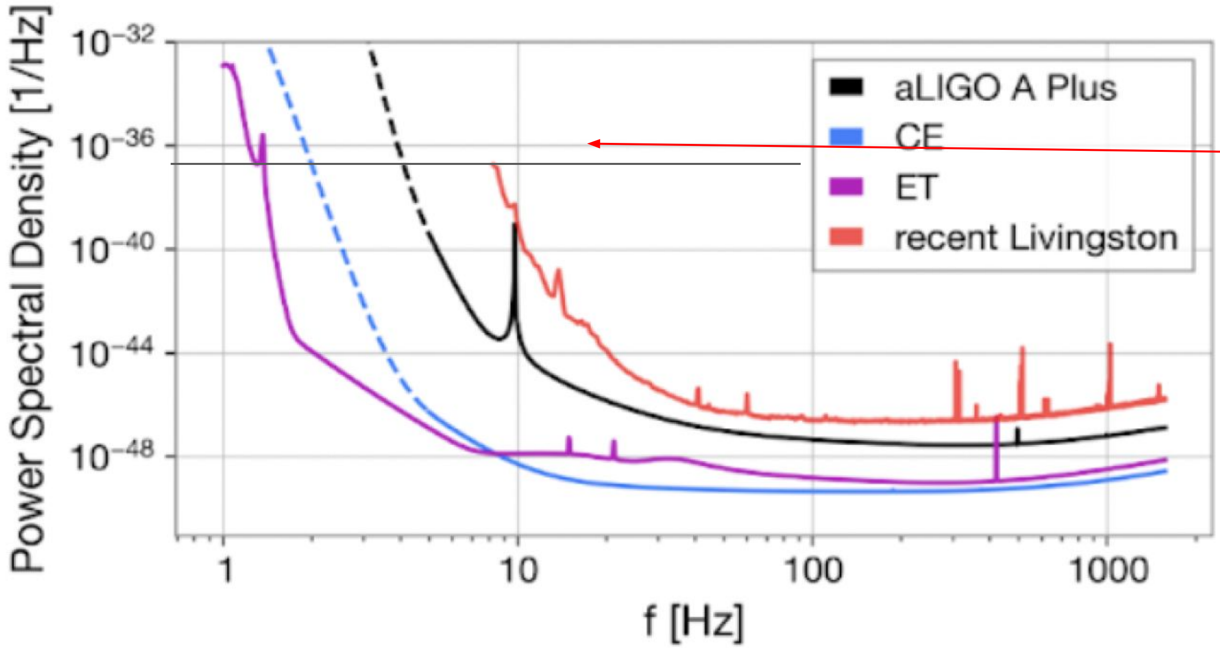
# Upcoming Detectors (CE, ET)




The experimental models of Cosmic Explorer (left) and the Einstein Telescope (right) are shown.

# Detector Noise Levels are Improving at Low Frequencies!




LIGO PSD at Cutoff Frequency (8Hz)

The PSDs of Upgraded LIGO, Cosmic Explorer, Einstein Telescope, and LIGO Livingston are compared. Lazarow-Leslie-Dai 2024.

# Project Goal

For long-lasting strain signals, the Earth’s rotation will have a significant effect. This is important for neutron star merger (minutes to hours) detection.

We want to determine the chirp mass threshold at which signals detected by next generation detectors are long enough such that the Earth’s rotation is no longer a negligible effect.

# The Strain in Detail (No Rotation)

$$h(t) = F_+ h_+(t - \Delta t) + F_\times h_\times(t - \Delta t)$$ time domain

FFT ↓ IFFT ↑

$$\tilde{h}(f) = [F_+ \tilde{h}_+(f) + F_\times \tilde{h}_\times(f)] e^{-2\pi i f \Delta t}$$ frequency domain

$F_+$ , $F_\times$ : **detector response coefficients** (constants in time **IF** neglecting rotation)

$\Delta t$ : **time delay** between detectors (constant in time **IF** neglecting rotation)

# Detector Response Coefficients

$$h(t) = \underline{F_+} h_+(t - \Delta t) + \underline{F_\times} h_\times(t - \Delta t)$$

$$F_+ = \vec{X}^\top \underline{D} \vec{X} - \vec{Y}^\top D \vec{Y} \qquad F_\times = \vec{X}^\top D \vec{Y} + \vec{Y}^\top D \vec{X}$$

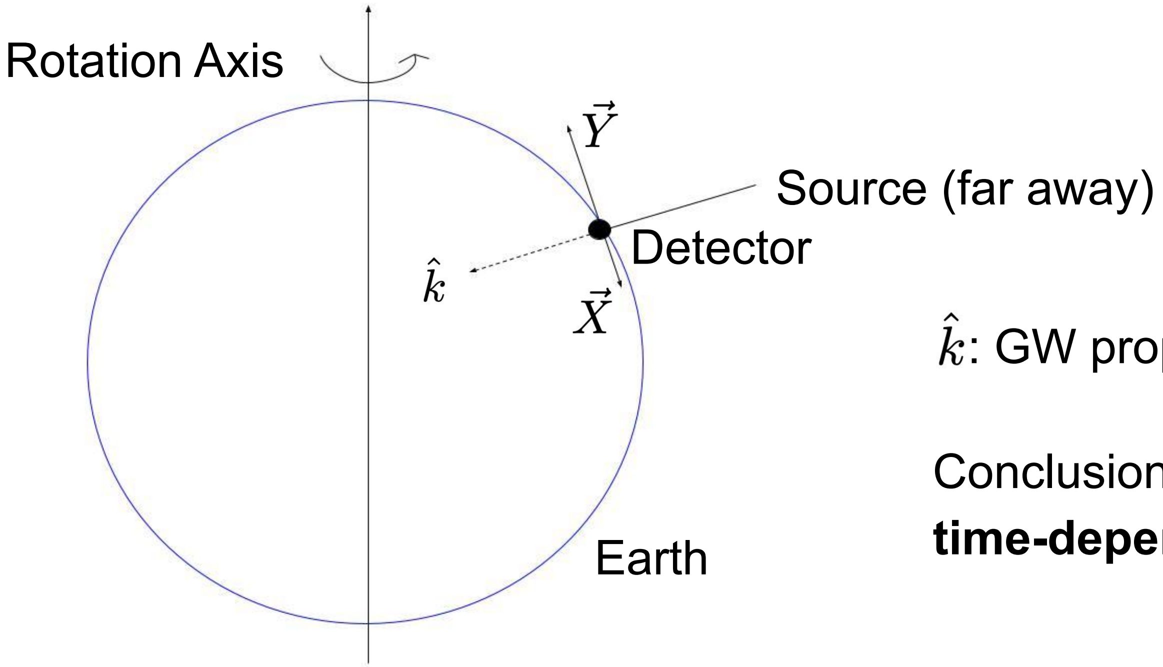
3 x 3, symmetric, constant matrix describing detector orientation, location

$$D = \begin{pmatrix} D_{11} & D_{12} & D_{13} \\ D_{12} & D_{22} & D_{23} \\ D_{13} & D_{23} & D_{33} \end{pmatrix}$$

# Detector Response Coefficients (Cont.)

$$F_{+} = \vec{X}^{\top} D \vec{X} - \vec{Y}^{\top} D \vec{Y} \qquad F_{\times} = \vec{X}^{\top} D \underline{\vec{Y}} + \vec{Y}^{\top} D \underline{\vec{X}}$$

Axes of wave-frame




$\hat{k}$: GW propagation direction

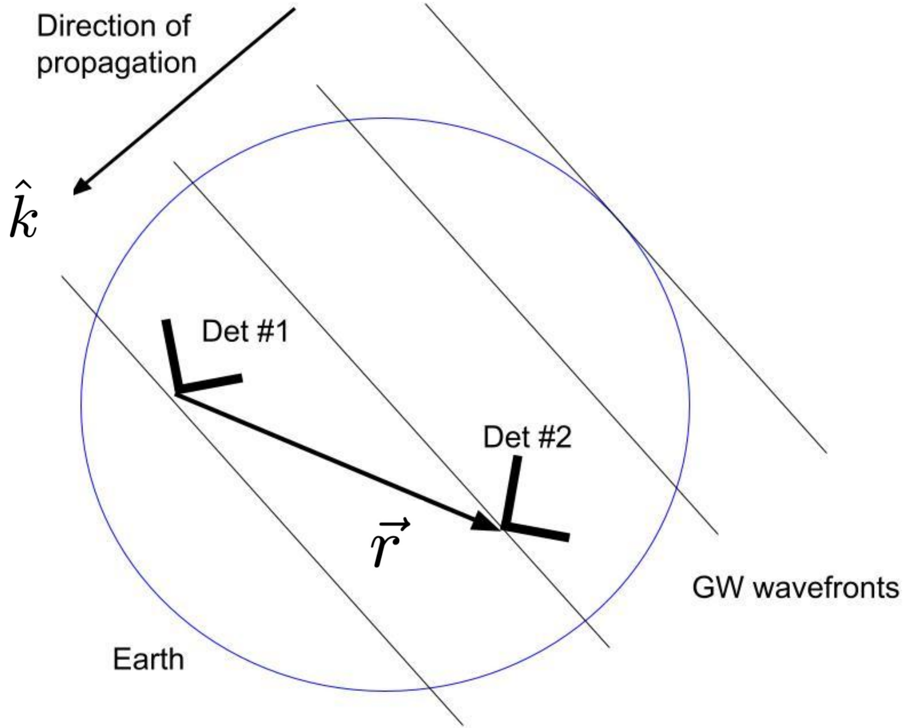
Conclusion: these quantities are **time-dependent**

# Time Delay

$$h(t) = F_+ h_+(t - \underline{\Delta t}) + F_\times h_\times(t - \Delta t)$$

Time delay between #1, #2:

$$\Delta t = \frac{1}{c}\vec{r} \cdot \hat{k}$$




Conclusion: the time delay is also **time-dependent**

ROTATING MODEL (BOARD WORK)

# Introducing a Time Dependence

The time-dependent strain in the time domain takes the form:

$$h_{\mathrm{exact}}(t) = F_+(t, \mathrm{det})h_+(t - t_{\mathrm{delay}}(t, \mathrm{det})) + F_\times(t, \mathrm{det})h_\times(t - t_{\mathrm{delay}}(t, \mathrm{det}))$$

Detector response coefficients no longer constant in time

Time delay no longer constant in time

# Derivation of First Order Rotating Earth Model

First order perturbative expansions in time:

$$F_+(t) = F_+(t_0) + \Omega \cdot (t - t_0) \cdot \gamma_+$$

$$F_x(t) = F_x(t_0) + \Omega \cdot (t - t_0) \cdot \gamma_x$$

$$\Delta t(t) = \Delta t(t_0) + \Omega \cdot (t - t_0) \cdot \eta$$

# Bayesian Inference and the Stationary Noise Approximation

$$P(\theta \mid d) \propto P(d \mid \theta) \cdot P(\theta) = \mathcal{L} \cdot \mathrm{Prior}$$

$$log\mathcal{L} = \langle d(f) | h(f) \rangle - \frac{1}{2} \langle d(f) | h(f) \rangle$$

# Next Steps: Testing Our Perturbative Model

1. Create an injection of mock data consisting of:
   a. Noise (drawn from Gaussian at each frequency)
   b. Model data (generated from a non-perturbative rotating model in time domain)

2. Test by doing parameter estimation on the non-rotating and rotating (perturbative) models.

3. Where does the accuracy of estimated parameters diverge for those two models?

# Acknowledgements

1. Lazarow, M., Leslie, N., & Dai, L. (2024). A Gravitational Waveform Model for Detecting Accelerating Inspiraling Binaries. arXiv. https://arxiv.org/abs/2401.04175

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